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# A Program for Generation of Synthetic Stratigraphic Sections

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ABSTRACT.—This report contains a simple computer program for the generation of synthetic stratigraphic sections. To generate a stratigraphic section, a transition procedure to go from one lithology to another is needed, plus the thickness distributions of the different lithologies. Dependent Markovian processes define the transition procedure. Several examples illustrate the program.

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## INTRODUCTION

Stratigraphic sections, be they cores from a modern carbonate basin, pre-Cambrian sequences of interbedded tuffs and basalt flows, or organic-rich sediments of a now abandoned meander bend of an alluviating river, are fundamental to all phases of geologic study. This report contains a simple computer program for the generation of synthetic stratigraphic sections. Applications of this program can be found in Potter and Blakely (1967 and 1968).

One might well ask, "Why bother to synthesize stratigraphic sections, when we have so many real ones to consider?" Close correspondence between real and synthetic sections suggests that the factors used in the synthesizing process may indeed correspond to those in nature and thus provide a check to an investigator's assumption, the underlying idea being that experimental duplication of reality is good evidence that we understand it. Simulation of individual sections is also an important first step toward synthesizing rock bodies such as sandstone or carbonate reservoirs or even portions of a sedimentary basin as has been done by Harbaugh (1966, figs. 15 to 18) or, for example, the delta of a large river system. For large geologic features only simulation by computer is generally practical as a means of modeling nature. The program that follows is offered as an early step in this direction. Other examples of simulation may be found in Sedimentation Seminar (1966), Bonham-Carter and Sutherland (1967), and Oertal and Walton (1968). Krumbein (1968) uses Markov processes to simulate cross sections of transgressive-regressive sedimentation.

THEORY

To generate a stratigraphic section two components are required: a transition procedure from one lithology to another and the thickness distributions of the different lithologies.

The transition from one lithology to another uses random processes, either independent or dependent ones. Dependent random processes, those with a "memory" such that past deposition has an influence on either present or future deposition, appear to be more relevant—at least to sedimentation processes—than "memoryless", independent ones. A random or probability process with a memory of one step is called a *Markov process.*

Central to Markov processes is the concept of *conditional probability,* which is defined as

$$P(A/B) = P\ (AB)/P(B)$$

where $P(A/B)$ is read "the probability of the event A given the event B" $P(AB)$ is read "the probability of A and B," their joint occurrence, and $P(B)$ is the probability of B above.

Another concept is that of a *state.* For example, the traditional division of all sediments into limestone, sandstone, and shale divides the sedimentary realm into three states. But in a carbonate problem an investigator might choose the states to be lithologies such as mudstone, micrite, micritic-bryozoan limestone, and bryzoan limestone. The possibilities are endless and depend only on the fineness of the lithologic classification judged necessary by the investigator. If the number of states recognized is finite, as is usually the case, the process is called a *finite Markov process.*

The transitions from one state to another in a Markov process are given by a transition matrix of probabilities. This transition matrix specifies the memory of the process. The individual elements $p_{ij}$ of a transition matrix are called transition probabilities and give the probability of the occurrence of state j, say mudstone, if the preceding state were i, say a micritic-bryozoan limestone. The probabilities are the "empirical probabilities," $N_{ij}/N_i$, where $N_{ij}$ is the number of pairs ij and $N_i$ is the total number of i's. Each $p_{ij}$ is a number between 0 and 1. If $p_{ij}$ is 0, the transition from state i to state j cannot occur; if $p_{ij}$ is 1, the transition is certain to occur. If $p_{ij} = 1$, the state i cannot be left.

With these definitions behind us, we may more formally say that a sequence of events forms a Markov process, if for any i, j, n- 1, 2, 3, . . . the probability that an event A will occur at a given trial, if specified events have occurred at the preceding event, but not on the other preceding events. Thus $P(A/B) = P(A/B, C, D, \ldots\ldots)$.

For a three state system of cross-bedding ($S_1$), ripple mark ($S_2$), and parting lineation ($S_3$), such as might define a quartzose sandstone body, the transition matrix P is

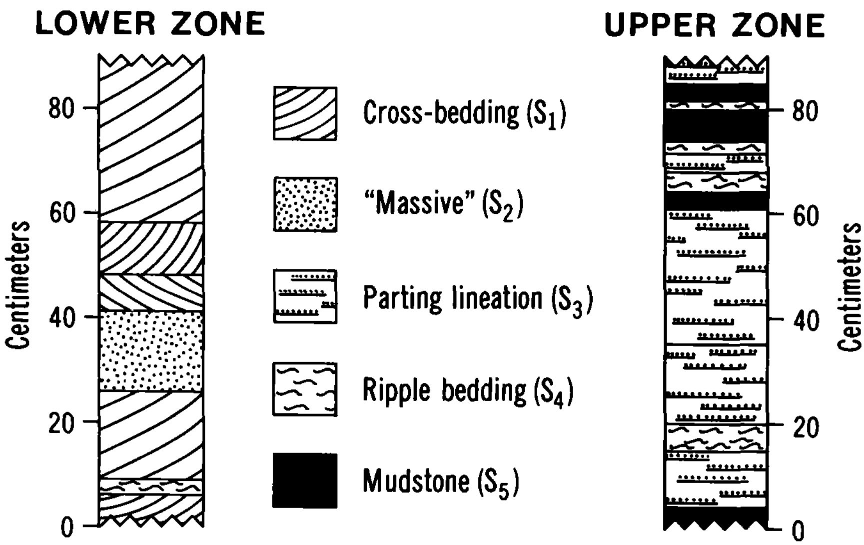
$$P = \begin{Bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{Bmatrix}$$

where $p_{11}$ is the probability of a cross-bed following a cross-bed, $p_{12}$ is the probability of ripple mark following a cross-bed, $p_{32}$ is the probability of ripple mark following parting lineation, etc. Each row of this matrix sums to one, since *something* always has to follow any given lithology. Additional information about Markov processes can be found in Kemeny, *et al.* (1959, p. 384-438), Kemeny and Snell (1960), Parzen (1962), and Fisz (1963, p. 250-334).

Like the individual transition probabilities, $p_{ij}$, thickness distributions can be determined by field observation or may be assumed. Usually thickness distributions are highly skewed toward the thinner beds and approximate a log normal distribution. Specifying the duration of a particular state by thickness rather than time (cf. Harbaugh, 1966) is useful, because thickness is directly measurable in cores and outcrops whereas time is not. Moreover, regression equations between thickness and other lithologic characters such as grain size, porosity, permeability, and others can be obtained.

## PROCEDURE

To illustrate the process, a stratigraphic section through a quartzose sand body of fluvial origin was generated. Potter and Blakely (1968) provide geologic details. The sand body was assumed to have five states: sand in the




$$\begin{array}{cc} \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \end{array} & \\ \left[\begin{array}{ccccc} .55 & .30 & .05 & .05 & .05 \\ .65 & .25 & .05 & .05 & .00 \\ .50 & .20 & .05 & .25 & .00 \\ .60 & .15 & .15 & .00 & .10 \\ .45 & .50 & .00 & .00 & .05 \end{array}\right] & \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{array} \end{array} \qquad \begin{array}{cc} \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \end{array} & \\ \left[\begin{array}{ccccc} .15 & .15 & .20 & .30 & .20 \\ .05 & .05 & .20 & .50 & .20 \\ .00 & .00 & .30 & .50 & .20 \\ .05 & .00 & .15 & .50 & .30 \\ .00 & .00 & .55 & .00 & .45 \end{array}\right] & \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{array} \end{array}$$

TEXT-FIGURE 1.—Details of lower and upper zones of synthetic sand body. Contrasting transition matrices control lithologic proportions of the two zones (Potter and Blakely, 1967, fig. 6).

form of four different sedimentary structures, plus mudstone. These states were cross-bedding ($S_1$), massive bedding ($S_2$), parting lineation ($S_3$), ripple bedding ($S_4$), and mudstone ($S_5$). The sand body was further assumed to have three zones: a lower dominantly cross-bedded zone, a middle zone in which parting lineation and mudstone were dominant, and an upper zone transitional to mudstone. The use of a different matrix, P, for each zone in effect alters the depositional process, because the transition matrix controls the proportions of the different states or lithologies that accumulate as deposition proceeds. Text-figure 1 shows two different transition matrices and the resulting proportions of bedding types in the lower and middle zones of the sand body. Transition probabilities for the three different zones and log normally distributed thicknesses for the different bedding types were assumed.

A computer program was written to synthesize stratigraphic sections (Table 1). The program also includes an optional provision to linearly reduce or expand the thickness of one or more of the different lithologies as deposition proceeds. For fluvial sands, bed thickness is a rough measure of the transport competence of stream discharge.

TABLE 1

Annotated Computer Program for Synthetic Stratigraphic Sections

| | | | |
|---|---|---|---|
| | 1. | TITLE Markov sedimentation with varying matrix | |
| | 2. | SUBSCRIPTS (I,J), (K,L), M, N, T, A | |
| | 3. | FORMAT ONE(S2DT), TWO(S4DP2DT), THREE(S3DT) | |
| | 4. | DATA THICK(5,20), PTHICK(5,20), R(5), KMAX(5), RAND(100), LITHP(5,5), X(100) | |
| | 5. | BEGIN | |
| | 6. | START: CARR(1) | |
| | 7. | LLIM=KEYBD–1 | type in number of lithologies |
| | 8. | CARR(1) | |
| | 9. | KLIM=5*LLIM | |
| | 10. | FOR M=O(1)LLIM BEGIN | |
| | 11. | CARR(1) | type in number of class intervals in each lithology |
| | 12. | PRINT (ONE)=M+1 | |
| | 13. | KMAX[M]=KEYBD–1 END | |
| | 14. | LAB1: CARR(1) | |
| | 15. | FOR I=0(1)LLIM BEGIN | |
| | 16. | M=I | |
| | 17. | LIMIT=KMAX[M]*5 | |
| | 18. | FOR J=0(5)LIMIT BEGIN | |
| 071 | 19. | CARR(1) | |
| | 20. | PRINT (ONE)=I+1 | type in the thickness and relative probabilities for each lithology |
| | 21. | PRINT (ONE)=J/5+1 | |
| | 22. | THICK[I, J]=KEYBD | |
| | 23. | PTHICK[I, J]=KEYBD/100 END | |
| | 24. | CARR(1) END | |
| | 25. | FOR I=0(1)LLIM BEGIN | |
| | 26. | M=I | |
| | 27. | K=I | |
| | 28. | LIMIT=KMAX[M]*5 | Accumulate the probabilities |
| | 29. | FOR J=5(5)LIMIT BEGIN | |
| | 30. | L=J–5 | |
| | 31. | PTHICK[I, J]=PTHICK[K, L]+PTHICK[I, J] END END | |
| 115 | 32. | LAB3: CARR(2) | |
| | 33. | FIRST=KEYBD–1 | type in the first lithology (arbitrary) |
| | 34. | CARR(1) | |
| | 35. | BOTTOM=KEYBD | type in the total thickness of all sections |

TABLE I (Continued)

```
    CARR(1)
    T=1                                          set tally
    DEPTH=O                                      set thickness
    A=O                                          set punch tally
130 LAB2: CARR(1)
    FOR M=O(1)LLIM BEGIN
    CARR(1)                                      type in the reduction coefficients
    PRINT (ONE)=M+1
    R[M]–KEYBD/100 END
    LOOP. CARR(1)
    FOR I=O(1)LLIM BEGIN
    FOR J=O(5)KLIM BEGIN
155 CARR(1)
    PRINT (ONE)=I+1                              type in the transition matrices
    PRINT (ONE)=J/5+1
    LITHP[I, J]=KEYBD/100 END
    CARR(1) END
    CARR(2)
    LAB4: RLIMIT=KEYBD                           type in the distance from the
    CARR(5)                                      bottom to the end of this section
    FOR I=O(1)LLIM BEGIN
    K=I
    FOR J=5(5)KLIM BEGIN
    L=J–5                                        accumulate transition matrix
    LITHP[I,J] = LITHP[K,L] + LITHP[I,J] END END
    I=FIRST
    OVER: READ (P) RAND
    FOR M=O(2)98 BEGIN                           start main computing loop
    PRINT (THREE) = T                            type serial number
    X[A]=T
    PRINT (ONE)=I+1                              type lithology
    X[A+1]=I+1
    LAB5: TEST=RAND[M]
    FOR J=O(5)KLIM BEGIN
    IF LITHP[I,J]>TEST                           find next lithology
    GO TO FOUND END
    FOUND: NLITH=J/5
    N=M+1
    LAB6: TEST=RAND[N]
    LIMIT=KMAX[I]*5
    FOR J=0(5)LIMIT BEGIN
    IF TEST<PTHICK[I,J]                          find thickness
    GO TO OUT END
    OUT: N=I
    NOW=THICK[I,J]*(1–R[N]* DEPTH/BOTTOM)        modify thickness
    PRINT (TWO)=NOW                              print thickness
    X[A+2]=NOW
    DEPTH=DEPTH+NOW
    PRINT (TWO)=DEPTH                            print depth
    X[A+3]=DEPTH
    PRINT (TWO)=DEPTH – NOW/2                    print center of unit
    X[A+4]=DEPTH–NOW/2
    A=A+5
    IF A > 95
    GO TO PUNCH
    LAB7: I=NLITH
    T = T + 1
    CARR(1)
    IF DEPTH > BOTTOM
    GO TO COMPLETE
```

TABLE I (Continued)

```
     96.  IF DEPTH > RLIMIT
     97.  GO TO LOOP END
     98.  GO TO OVER
     99.  COMPLETE: WRITE (P) X                    final step
    100.  STOP
    101.  GO TO START
314 102.  FLUB: I=KEYBD—1                          error correction section
    103.  J=(KEYBD—1)*5
    104.  HALT: STOP
    105.  GO TO HALT
    106.  PUNCH: WRITE (P) X
    107.  A=0
    108.  LAST: GO TO LAB7
    109.  END
```

Sections are generated by the following procedure:

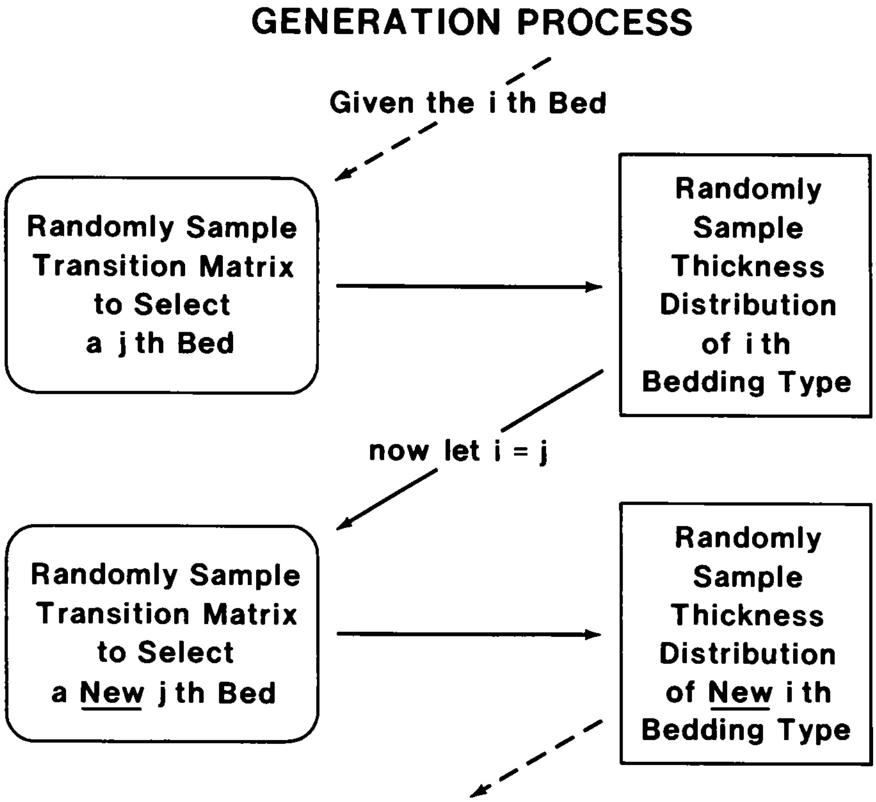
1) Initial state, i, is chosen randomly; this specifies a particular row of the transition matrix.
2) The thickness, t, is then chosen randomly from the frequency distribution of the ith state.
3) Convert t to t′, where t′ may either be greater or less than t.
4) The following state, j, is selected in accordance with the probabilities of the $p_{ij}$'s of the ith row.
5) With state, thickness, and overlying state now selected for the first bed, let j specify the state i for the new bed, 2(in short, let $i = j$).
6) Return to step 2.

The accompanying program (Table 1) is written in Algo, a language for the Control Data Corporation G-15 computer. No special subroutines are necessary for this program. It contains 108 steps. Side comments describe its various parts. This program, outlined by the flow diagram of Text-figure 2, may be reentered at any of the circled points and thus has maximum flexibility. For example, the thickness reduction coefficients may be altered for the next stratigraphic section by entering the program at step 2 (40. Lab. 2) of Table I. Program step 102 begins an error correcting section from which one can manually transfer back to either steps 19 or 48 of Table 1.

The core of the program is an algorithm, a computational procedure, for choosing the next lithology and thickness (Steps 63 through 97). This algorithm is best described by an example. Suppose the current lithology is a massive bed, $S_2$, and the transition matrix P is

$$P = \begin{Bmatrix} 0.10 & 0.40 & 0.25 & 0.15 & 0.10 \\ 0.15 & 0.42 & 0.28 & 0.10 & 0.05 \\ 0.15 & 0.35 & 0.30 & 0.15 & 0.05 \\ 0.10 & 0.50 & 0.20 & 0.15 & 0.05 \\ 0.10 & 0.50 & 0.25 & 0.15 & 0.00 \end{Bmatrix}$$

The second row of the matrix gives the probability of a massive bed being followed by each of the five bedding types. Now successively cumulate the



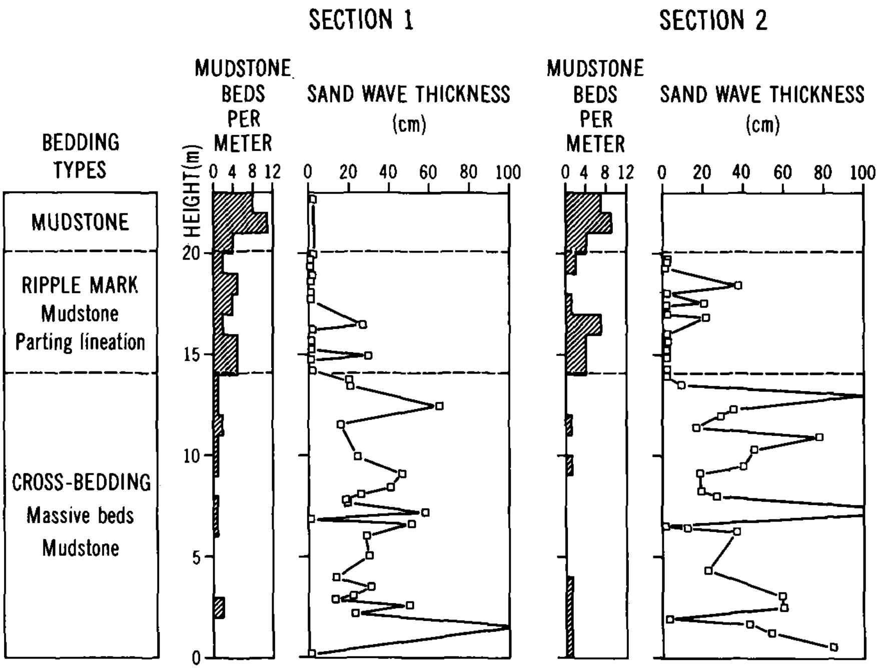

TEXT-FIGURE 2.—Flow diagram of computer program for synthesis of stratigraphic sections.

probabilities in the second row to obtain 0.15, 0.57, 0.85, 0.95, and 1.00. A two digit, random number can now be used to select the next lithology. For example, if the random number were 0.13, the next lithology would be a cross-bed; if it were 0.56, it would be another massive bed; and if it were 0.95, a rippled bed would be chosen. In short, the cumulative probabilities provide class limits which determine, for a given random number, which lithology will follow a preceding lithology. The selection is made randomly, but in accordance with the transition probabilities for the second row. A similar procedure is followed for the other rows.

An analogous algorithm is used to select randomly a thickness from the thickness distribution of a particular lithology.

## RESULTS

Text-figure 3 shows two synthetic profiles of a fluvial sand body. The left-hand column indicates the dominant bedding type of each of the three zones. The second column indicates the frequency of thin mudstones per meter; such




TEXT-FIGURE 3.—Two synthetic sections of a fluvial sand body. Each section generated with same transition matrices and thickness distributions. Contrast in mudstones per meter and position and thickness of ripple mark and cross-bedding result from random element in generation process. Principal bedding type of different zones in bold face.

mudstones are important vertical barriers to fluid flow in a reservoir. The third column shows an irregular upward decline in thickness of cross-bedding and ripple marks as is found in many fluvial sand bodies.

Each of the two sections was generated using the same transition matrices and thickness distributions and consequently both are generally similar. Because of the random element—one that surely occurs in every depositional process—the details of corresponding zones in each section differ. This is most clearly displayed in Table 2 which shows part of the data output for the

TABLE 2

Two segments of computer output for basal zone of sand body

| *Ident* | *Lith* | *Thick* | *Height* | *Center* | |
|---|---|---|---|---|---|
| 1 | 3 | 22.50 | 22.50 | 11.25 | |
| 2 | 4 | 1.00 | 23.50 | 23.00 | Output of the bottom section |
| 3 | 5 | 18.50 | 42.00 | 32.75 | |
| 4 | 2 | 25.00 | 67.00 | 54.50 | |
| 5 | 5 | 30.50 | 97.50 | 82.25 | |
| 6 | 1 | 111.59 | 209.09 | 153.29 | |

TABLE 2 (Continued)

| | | | | | |
|---|---|---|---|---|---|
| 7 | 1 | 23.40 | 232.50 | 220.79 | |
| 8 | 1 | 51.10 | 283.61 | 258.05 | |
| 9 | 1 | 13.70 | 297.31 | 290.46 | Section one |
| 10 | 1 | 22.73 | 320.05 | 308.68 | |
| 11 | 5 | 2.50 | 322.55 | 321.30 | |
| 12 | 5 | 10.50 | 333.05 | 327.80 | |
| 13 | 1 | 31.45 | 364.50 | 348.77 | |
| | | 25.00 | 389.50 | 377.00 | |
| | | | 402.72 | 396.11 | |
| 8 | | | | 445.22 | |
| 9 | 2 | | | | |
| 10 | 5 | 10.50 | | | |
| 11 | 2 | 55.00 | 425.09 | | |
| 12 | 1 | 21.76 | 446.86 | | |
| 13 | 5 | 22.50 | 469.36 | 458.11 | |
| 14 | 2 | 15.00 | 484.36 | 476.86 | |
| 15 | 2 | 85.00 | 569.36 | 526.86 | |
| 16 | 2 | 15.00 | 584.36 | 576.86 | |
| 17 | 2 | 25.00 | 609.36 | 596.86 | |
| 18 | 1 | 36.65 | 646.01 | 627.68 | |
| 19 | 1 | 12.05 | 658.06 | 652.04 | |
| 20 | 4 | 1.00 | 659.06 | 658.56 | |
| 21 | 1 | 131.90 | 790.97 | 725.01 | |
| 22 | 1 | 26.57 | 817.54 | 804.25 | |
| 23 | 1 | 18.78 | 836.32 | 826.93 | |
| 24 | 2 | 55.00 | 891.32 | 863.82 | |
| 25 | 3 | 18.50 | 909.82 | 900.57 | Section two |
| 26 | 1 | 18.07 | 927.90 | 918.86 | |
| 27 | 1 | 39.46 | 967.36 | 947.63 | |
| 28 | 5 | 10.50 | 977.86 | 972.61 | |
| 29 | 2 | 35.00 | 1012.90 | 995.36 | |
| 30 | 1 | 44.96 | 1057.80 | 1035.30 | |

two basal zones. These contrasts, arising solely from the random sampling of the same transition matrices and thickness distributions, are also well displayed graphically in Text-figure 3.

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